PROBLEM 2.8

KNOWN:

$$y(t) = 25 + 10\sin 6\pi t$$

FIND: $\bar{y}$ and y_{rms} for the time periods t_1 to t_2 listed below

a) 0 to 0.1 sec
b) 0.4 to 0.5 sec
c) 0 to 1/3 sec
d) 0 to 20 sec

SOLUTION:

For the continuous function $y(t)$, the average may be expressed

$$\bar{y} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} y(t)\,dt$$

and the rms as

$$y_{rms} = \sqrt{\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} \left[y(t)\right]^2 dt}$$

For $y(t) = 25 + 10\sin 6\pi t$, the average is given by

$$\begin{aligned}\bar{y} &= \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} \left(25 + 10\sin 6\pi t\right) dt \\ &= \frac{1}{t_2 - t_1}\left[25t - \frac{10}{6\pi}\cos 6\pi t\Big|_{t_1}^{t_2}\right] \\ &= \frac{1}{t_2 - t_1}\left[25\left(t_2 - t_1\right) - \frac{10}{6\pi}\left(\cos 6\pi t_2 - \cos 6\pi t_1\right)\right]\end{aligned}$$

and the rms as

$$\begin{aligned}y_{rms} &= \left\{\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} \left(25 + 10\sin 6\pi t\right)^2 dt\right\}^{1/2} \\ &= \left\{\frac{1}{t_2 - t_1}\left[625t - \frac{500}{6\pi}\cos 6\pi t + 100\left(\frac{-1}{12\pi}\sin 6\pi t\cos 6\pi t + \frac{1}{2}t\right)\right]\Bigg|_{t_1}^{t_2}\right\}^{1/2}\end{aligned}$$

The resulting numerical values are

a) $\overline{y} = 25.69 \qquad y_{rms} = 31.85$

b) $\overline{y} = 25.69 \qquad y_{rms} = 31.85$

c) $\overline{y} = 25 \qquad y_{rms} = 25.98$

d) $\overline{y} = 25 \qquad y_{rms} = 25.98$

COMMENT: The average and rms values for the time period 0 to 20 seconds represents the long-term average behavior of the signal. The result in parts a) and b) are accurate over the specified time periods and for a measured signal may have specific significance. The period 0 to 1/3 represents one complete cycle of the simple periodic signal and results in average and rms values which accurately represent the long-term behavior of the signal.

PROBLEM 2.9

KNOWN: (a) $y(t) = 3t$ for $0 \le t \le 2$ s with $T = 2$ s
(b) $y(t) = 1.5t$ [V] for $0 \le t \le 2$ s and
$y(t) = 0$ [V] for $2 \le t \le 4$ with $T = 4$ s

FIND: $\overline{y}$, y_{rms}

SOLUTION

(a) The average (or mean) value of a continuous function is given by

$$\overline{y} = \frac{\int_{t_1}^{t_2} y(t)dt}{\int_{t_1}^{t_2} dt} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} 3t dt = \frac{1}{T-0}\int_0^T 3t dt = \frac{1}{T}\frac{3t^2}{2}\bigg|_0^T = \frac{1}{2}\frac{3t^2}{2}\bigg|_0^2 = 3V$$

$$y_{rms} = \sqrt{\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} y(t)^2 dt} = \sqrt{\frac{1}{T-0}\int_0^T 9t^2 dt} = \sqrt{\frac{1}{2}\frac{9t^3}{3}\bigg|_0^2} = 3.46V$$

(b) The average (or mean) value of a continuous function is given by

$$\overline{y} = \frac{\int_{t_1}^{t_2} y(t)dt}{\int_{t_1}^{t_2} dt} = \frac{1}{4-0}\left[\int_0^2 1.5t dt + \int_2^4 0 dt\right] = \frac{1}{4}\int_0^2 1.5t dt = \frac{1}{4}\frac{1.5t^2}{2}\bigg|_0^2 = \frac{6}{8} = 0.75V$$

$$y_{rms} = \sqrt{\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} y(t)^2 dt} = \sqrt{\frac{1}{4}\left[\int_0^2 (1.5t)^2 dt + \int_2^4 (0)^2 dt\right]} = \sqrt{\frac{1}{4}\frac{2.25t^3}{3}\bigg|_0^2} = \sqrt{\frac{3}{2}} = 1.22V$$

COMMENT Keep in mind that the average and rms values are averaged over the entire signal period even if part of that period the signal is zero.

PROBLEM 2.10

KNOWN: *y(t) = 2sin 2πt*

FIND: $\overline{y}$, y_{rms} for different signal intervals

SOLUTION

We note that this signal has a frequency of 1 Hz (i.e., *2πf = 2π,* if *f* = 1). Hence, the signal has a period of *T = 1/f* = 1s.

The average (or mean) value is given by

$$\overline{y} = \frac{\int_{t_1}^{t_2} y(t)dt}{\int_{t_1}^{t_2} dx} = \frac{1}{t_2 - t_1}\left[\int_{t_1}^{t_2} y(t)dt\right]$$

(a) for 0 ≤ t ≤ 0.5 s

$$\overline{y} = \frac{1}{0.5-0}\left[\int_0^{0.5} 2\sin 2\pi t dt\right] = -\frac{2}{\pi}\cos 2\pi t\Big|_0^{0.5} = \frac{4}{\pi}V$$

(b) for 0 ≤ t ≤ 1

$$\overline{y} = \frac{1}{1-0}\left[\int_0^{1} 2\sin 2\pi t dt\right] = -\frac{2}{2\pi}\cos 2\pi t\Big|_0^{1} = 0V$$

(c) for 0 ≤ t ≤ 10

$$\overline{y} = \frac{1}{10-0}\left[\int_0^{10} 2\sin 2\pi t dt\right] = -\frac{2}{20\pi}\cos 2\pi t\Big|_0^{10} = 0V$$

The rms value is given by

$$y_{rms} = \sqrt{\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} y(t)^2 dt}$$

$$y_{rms} = \sqrt{\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} (2\sin 2\pi t)^2 dt} = \sqrt{\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} (4\sin^2 2\pi t) dt}$$

$$= \sqrt{\frac{4}{t_2 - t_1}\left(\frac{t}{2} - \frac{\sin 4\pi t}{8\pi}\right)\Bigg|_{t_1}^{t_2}}$$

(a) so for 0 ≤ *t* ≤ 0.5 s

$$y_{rms} = \sqrt{\frac{4}{0.5-0}\left(\frac{t}{2} - \frac{\sin 4\pi t}{8\pi}\right)\Bigg|_0^{0.5}} = \sqrt{2} = \frac{2}{\sqrt{2}}$$

(b) for $0 \le t \le 1$ s

$$y_{rms} = \sqrt{\frac{4}{1-0}\left(\frac{t}{2} - \frac{\sin 4\pi t}{8\pi}\right)\Bigg|_0^{1}} = \sqrt{2} = \frac{2}{\sqrt{2}}$$

(c) for $0 \le t \le 10$ s

$$y_{rms} = \sqrt{\frac{4}{10-0}\left(\frac{t}{2} - \frac{\sin 4\pi t}{8\pi}\right)\Bigg|_0^{10}} = \sqrt{2} = \frac{2}{\sqrt{2}}$$

PROBLEM 2.11

KNOWN: Discrete sampled data, corresponding to measurement every 0.4 seconds, as shown below.

t	$y_1(t)$	$y_2(t)$	t	$y_1(t)$	$y_2(t)$
0	0	0			
0.4	11.76	15.29	2.4	−11.76	−15.29
0.8	19.02	24.73	2.8	−19.02	−24.73
1.2	19.02	24.73	3.2	−19.02	−24.73
1.6	11.76	15.29	3.6	−11.76	−15.29
2.0	0	0	4.0	0	0

FIND: The mean and rms values of the measured data.

SOLUTION:

For a discrete signal the mean and rms are given by

$$\bar{y} = \frac{1}{N}\sum_{t=0}^{N-1} y_i \qquad\qquad y_{\text{rms}} = \sqrt{\frac{1}{N}\sum_{i=0}^{N-1} y_i^2}$$

The mean value for y_1 is 0 and for y_2 is also 0.

However, the rms value of y_1 is 13.49 and for y_2 is 17.53.

COMMENT: The mean value contains no information concerning the time varying nature of a signal; both these signals have an average value of 0. But the differences in the signals are made apparent when the rms value is examined.

PROBLEM 2.12

KNOWN: The effect of a moving average signal processing technique is to be determined for the signal in Figure 2.22 and $y(t) = \sin 5t + \cos 11t$

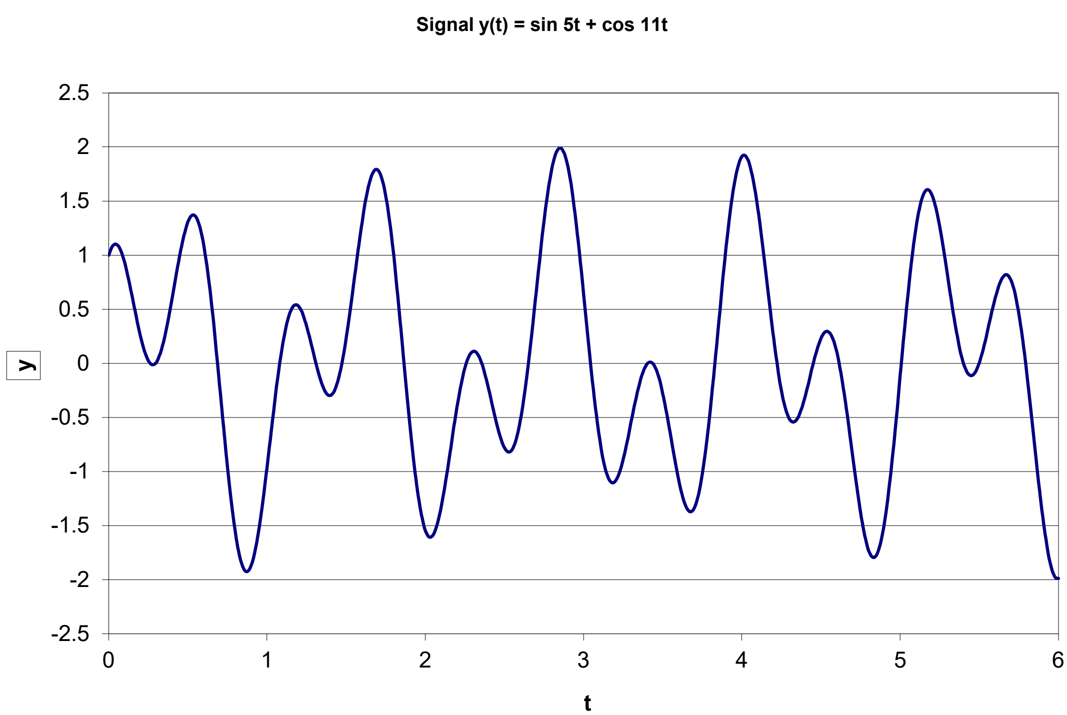
FIND: Discuss Figure 2.23 and plot the signal resulting from applying a moving average to $y(t)$.

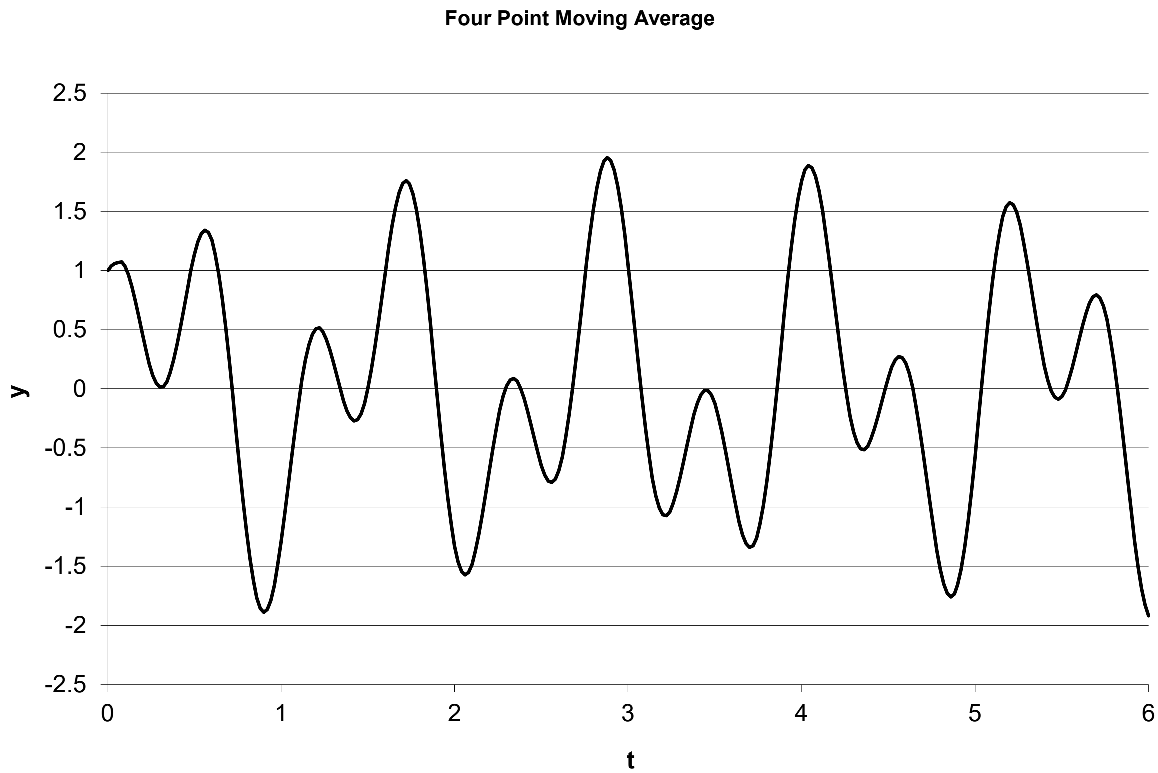
ASSUMPTIONS: The signal $y(t)$ may be represented by making a discrete representation with $\delta t = 0.05$.

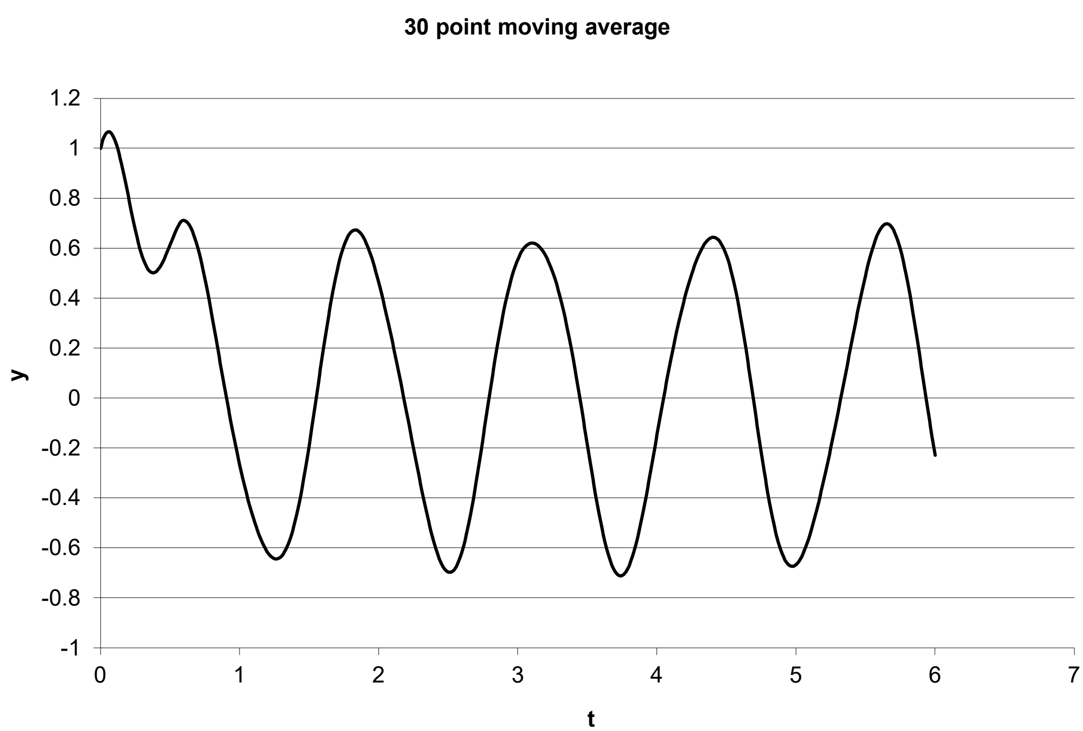
SOLUTION:

a) The signal in Figure 2.23 clearly has a reduced level of high frequency content as compared to that of Figure 2.22. In essence, this emphasizes longer-term (low frequency) variations while removing shorter-term (high) fluctuations. It is clear that the peak-to-peak value in the original signal (Figure 2.22) is significantly higher than in the signal that has been averaged (Figure 2.23) as the higher frequency information imposed on the lower frequency is averaged (filtered) away.

b) The figures below show in the effect of applying a moving average to $y(t) = \sin 5t + \cos 11t$.




Four Point Moving Average
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
y
0
1
2
3
4
5
6
t


30 point moving average
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
y
0
1
2
3
4
5
6
7
t